South Texas Project Risk-Informed GSI-191 Evaluation

Volume 3

Sump Temperature as a Function of Time and Break Size

Document: STP-RIGSI191-V03.07
Revision: 2
Date: January 17, 2013

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1. Summary of Analysis

Using simulated data from Texas A&M's RELAP-MELCOR simulation model, we develop a method that uses a piecewise least-squares fit and interpolation to generate time series profiles that represent the minimum, nominal, and maximum temperatures in the sump as a function of break size. The method allows for all break sizes in the interval [6", 27.5"], and any profile for that break size that lies between the minimum and maximum temperature profiles. We use simulated data from RELAP-MELCOR until 10 hours after a break. Our interpolations are limited to a time interval between 1772 seconds after the break (about the time recirculation begins for the largest break sizes we consider) and 10 hours. In implementation in CASA Grande it is necessary to extrapolate temperature profiles beyond 10 hours to 30 days after the break. For this extrapolation we recommend a linear fit from the temperature of our fits at 10 hours to a minimum temperature of 86 F in the cooling water system at 30 days. That said, we limit our discussion in this document to the model fits up to the 10-hour mark.

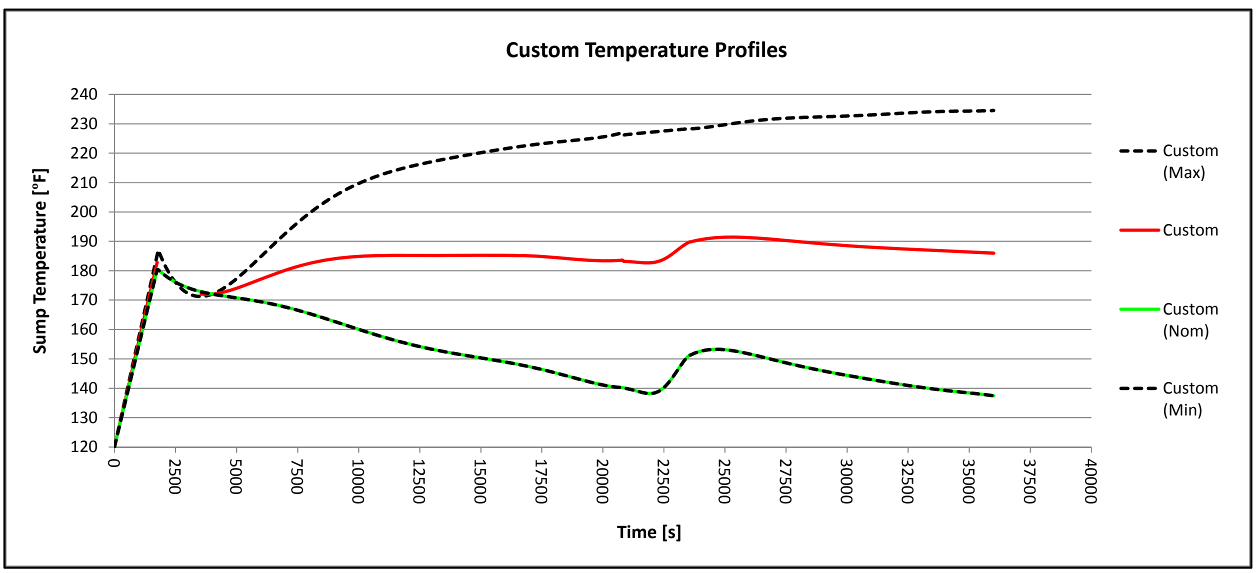

Figure 1. Custom Temperature Profile for a 10" LOCA, Located Halfway Between the Nominal and Maximum Profiles

2. Background and Data

Texas A&M provided simulated data from the RELAP-MELCOR simulation model for the following four LOCA break sizes: 6", 8", 15", and 27.5". They provided simulated data for the nominal temperature profile for all four break sizes, data for the maximum temperature profile for all four break sizes, and data for the minimum profile for only a 6" break. Table 1 summarizes this information.

Table 1. Summary of RELAP-MELCOR Simulated Data

Break Size/ Profile	6"	8"	15"	27"
Minimum	X			
Nominal	X	X	X	X
Maximum	X	X	X	X

The data provided are time series data, where both the time and the temperature were provided for about 2500 data points. The goal of this analysis is to develop a method for generating minimum, nominal, and maximum temperature profiles for any break size in the interval [6", 27.5"], and to be able to generate any profile in between the minimum and maximum profiles for any break size in the given interval.

3. Methodology

Our approach is to fit piecewise continuous polynomial functions to each of the given profiles for each break size, and then interpolate the parameters of those functions for all break sizes and profiles in between. For each time series provided, we partition time into four slices, which we denote $\{T_1, T_2, T_3, T_4\}$, where T_1 = [0, 0.492 hour], T_2 = (0.492, 5.77 hour], T_3 = (5.77, 6.54 hour], and T_4 = (6.54, 10 hour].

This study is primarily concerned with temperature in the sump after recirculation, which occurs after approximately 30 minutes (specifically, we use 1772 seconds). The first half hour of simulated data (T_1) has significant oscillations due to the fact that mass- and energy transfer to the containment system, via the break, is very unstable due to the associated phase change and further complicating factors such as actuation of spray systems and fan coolers in containment. In order to generate a complete profile, we simply model temperature as a linear function of time for about the first half hour, even though our focus is on the time period after recirculation begins. In the subsequent three time slices associated with recirculation, we assume temperature follows a polynomial function of degree six. A six-degree polynomial has a relatively large order. However, to capture the complex behavior of the simulated data in the tails of the 6" and 8" breaks, it was necessary to create three new time slices (after the linear time period T_1) as well as use polynomials of degree six. After approximately 7 hours, a recirculation event causes a jump in temperature, and in order to capture a jump that far in the tail and capture the curvature, we again needed polynomial functions of degree six and different parameters for those functions at time slices $\{T_2, T_3, T_4\}$. Our methodology involves interpolating parameters to form a new "custom" polynomial functions given polynomial functions at the simulated break sizes and minimum, nominal, or maximum conditions (profiles). Hence this method requires that all of the polynomials in a time slice be of the same degree. We use a six-degree polynomial function for all three time slices $\{T_2, T_3, T_4\}$ and for all break sizes and profiles. A six-degree polynomial has the form:

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4 + \theta_5 x^5 + \theta_6 x^6, \quad (1)$$

where y denotes temperature (degrees Fahrenheit) and x denotes time (seconds).

After fitting a polynomial function of the form of equation (1) to the data for each time slice $\{T_2, T_3, T_4\}$ for all break sizes and profiles, we can linearly interpolate the coefficients θ_i, $i = 0,...,6$, using the neighboring break size polynomial functions. The methodology is outlined as follows:

Step 1: Fit polynomial functions to each of the simulated data sets

Step 2: For a custom break size, determine the location fraction (L)

$$L = \frac{x-a}{b-a} \quad (2)$$

where:

a = Next Smallest Break Size (inches)

b = Next Largest Break Size (inches)

x = Custom Break Size (inches)

Step 3: For a custom break size, determine the new coefficients θ_i', $i = 0,...,6$, using the neighboring coefficients $(\theta_{B1}, \theta_{B2})$ for each of the three time slices $\{T_2, T_3, T_4\}$ via

$$\theta'_i = (1-L) * \theta_{B1,i} + L * \theta_{B2,i}, \quad i = 0,...,6. \quad (3)$$

These new coefficients define a six-degree polynomial function for any custom break size. By interpolating between "neighboring coefficients" we mean that equation (3) takes a convex combination of the coefficients of equation (1) for the largest break size smaller than the custom break size (*B1*) and for the smallest break size larger than the custom break size (*B2*).

We separately fit equation (1) to simulated data corresponding to the minimum profile, the nominal profile, and the maximum profile for each break size in Table 1. In what we describe above, when carrying out the procedure in steps 1-3, we must choose which profile to use. So we can obtain equation (1) for a 10.5" break under the nominal profile by interpolating the coefficients of the 8" and 15" break sizes for the nominal profile. We can repeat the same procedure to obtain a 10.5" break for the maximum profile. Suppose we have the coefficient for both of these fits in hand. We can then perform one more interpolation that is between the nominal profile and the maximum profile, and we do so using a *profile index,* which we denote *P*. With *P=1/2* we obtain an interpolation half-way between the nominal and maximum profiles for the 10.5" break. The profile index *P* is continuous on the interval [-1, 1] and reference points are as follows:

$$P = \begin{cases} -1 & \text{Use Minimum Profile} \\ 0 & \text{Use Nominal Profile} \\ 1 & \text{Use Maximum Profile.} \end{cases}$$

Values of *P* between (-1, 0) result in a profile that is a weighted average of the minimum profile and the nominal profile, values between (0, 1) result in a profile that is a weighted average of the nominal profile and the maximum profile, and values of -1, 0, and 1 produce the minimum, nominal, and maximum profiles respectively. The notion of a *weighted average* typically refers to a weighted arithmetic mean, but other forms of weighted means are also calculated. The concept of weighted averages can be extended to functions [1], and weighted averages of functions play an important role in the systems of weighted differential and integral calculus [2].

In summary, after selecting a custom break size and a value of the profile index *P*, this method produces the appropriate temperature profile by weighting the parameters from equations for neighboring break sizes and neighboring profiles.

4. Fitting Results

Tables 2, 4, and 6 present the results of a least-squares fit for all break sizes and the nominal, maximum, and minimum profiles respectively. We present the parameters of each of the linear functions used during time slice $T_{1,}$ and the parameters of each of the polynomial functions used during time slices T_2, T_3, and T_4. Tables 3, 5, and 7 present the R^2 goodness-of-fit values for the polynomial fits for all break sizes and the nominal, maximum, and minimum profiles, respectively. Figures 2 and 3 are plots of the data and the fitted functions for selected nominal profiles, Figures 5 and 6 for selected maximum profiles, and Figure 8 for the 6" minimum profile. The plots restrict attention to simulated data and fits after the first 1772 seconds (about 30 minutes). In order to obtain smooth transitions between neighboring time slices, we included a short warm-up and/or warm-down period with the data outside the slice of interest. As a result in some of the figures the fitted line "carries over" slightly into neighboring pieces in some cases. This is simply an artifact of the plotting procedure and does not affect the actual fit. Figures 4 and 7 are plots of the fit functions for all of the break sizes for the nominal and maximum profiles. (There is only one break size for the minimum profile and so we do not replicate the information in Figure 8.) Note that in Figure 4 for the nominal temperature profiles, the temperature decreases over time while in Figure 7 for the maximum temperature profiles, the temperature is still increasing after 10 hours. The nominal and maximum profiles are distinguished by the fact that the nominal case assumes access to the "full heat-removal system" while the maximum-profile case assumes only a limited heat-removal system. So in the former case, the heat removal rate exceeds the heat production rate, and the opposite occurs in the maximum-profile case.

Table 2. Fitting Results for Nominal Temperature Profiles

Function	Par.	Coefficients			
		6" Break	**8" Break**	**15" Break**	**27.5" Break**
Startup (T_1)	θ_0	0.013694286	0.014308571	0.017865714	0.019617143
	θ_1	120.00	120.00	120.00	120.00
Function 1 (T_2)	θ_0	206.460	214.460	187.685	152.898
	θ_1	-0.024	-0.034	-0.001	0.031
	θ_2	5.4023E-06	9.4859E-06	1.2445E-07	-9.2927E-06
	θ_3	-5.5721E-10	-1.3083E-09	-1.5989E-10	1.1288E-09
	θ_4	2.7794E-14	9.1334E-14	2.1284E-14	-6.9472E-14
	θ_5	-6.3980E-19	-3.1388E-18	-1.0532E-18	2.1287E-18
	θ_6	5.0829E-24	4.2187E-23	1.8191E-23	-2.5767E-23
Function 2 (T_3)	θ_0	-59035992.98	13545339.96	46133222.92	29096266.03
	θ_1	16269.17280	-3456.31260	-12649.42730	-8040.26920

	θ_2	-1.86675E+00	3.65109E-01	1.44409E+00	9.25007E-01
	θ_3	1.14152E-04	-2.04170E-05	-8.78594E-05	-5.67107E-05
	θ_4	-3.92349E-09	6.36641E-10	3.00452E-09	1.95413E-09
	θ_5	7.18661E-14	-1.04784E-14	-5.47552E-14	-3.58824E-14
	θ_6	-5.48049E-19	7.09621E-20	4.15460E-19	2.74309E-19
Function 3 (T_4)	θ_0	-254239.875	-131133.94	54347.56810	21204.831
	θ_1	51.18680	25.97370	-10.94880	-4.64160
	θ_2	-4.27275E-03	-2.13339E-03	9.16309E-04	4.21932E-04
	θ_3	1.89431E-07	9.31510E-08	-4.06793E-08	-2.02540E-08
	θ_4	-4.70515E-12	-2.28088E-12	1.01057E-12	5.41548E-13
	θ_5	6.20870E-17	2.96982E-17	-1.33232E-17	-7.64935E-18
	θ_6	-3.40068E-22	-1.60653E-22	7.28440E-23	4.46094E-23

Table 3. R^2 Values for Nominal Temperature Profile Data Fitting

Break Size / R^2	**T_2**	**T_3**	**T_4**
6"	0.9948	0.9916	0.9995
8"	0.9937	0.9950	0.9993
15"	0.9999	0.9384	0.9935
27.5"	0.9998	0.9126	0.9949
Mean	0.9971	0.9594	0.9968
Min	0.9937	0.9126	0.9935

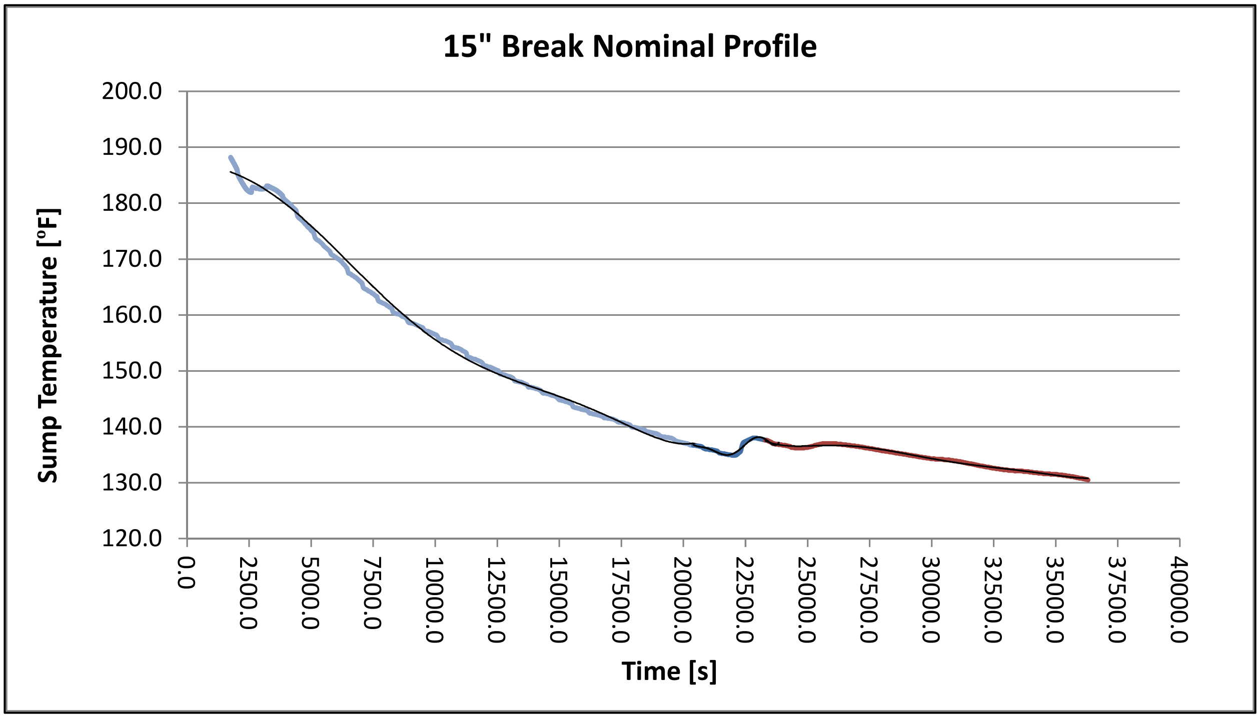

Figure 2. 15" Break Nominal Profile Data and Fitted Functions

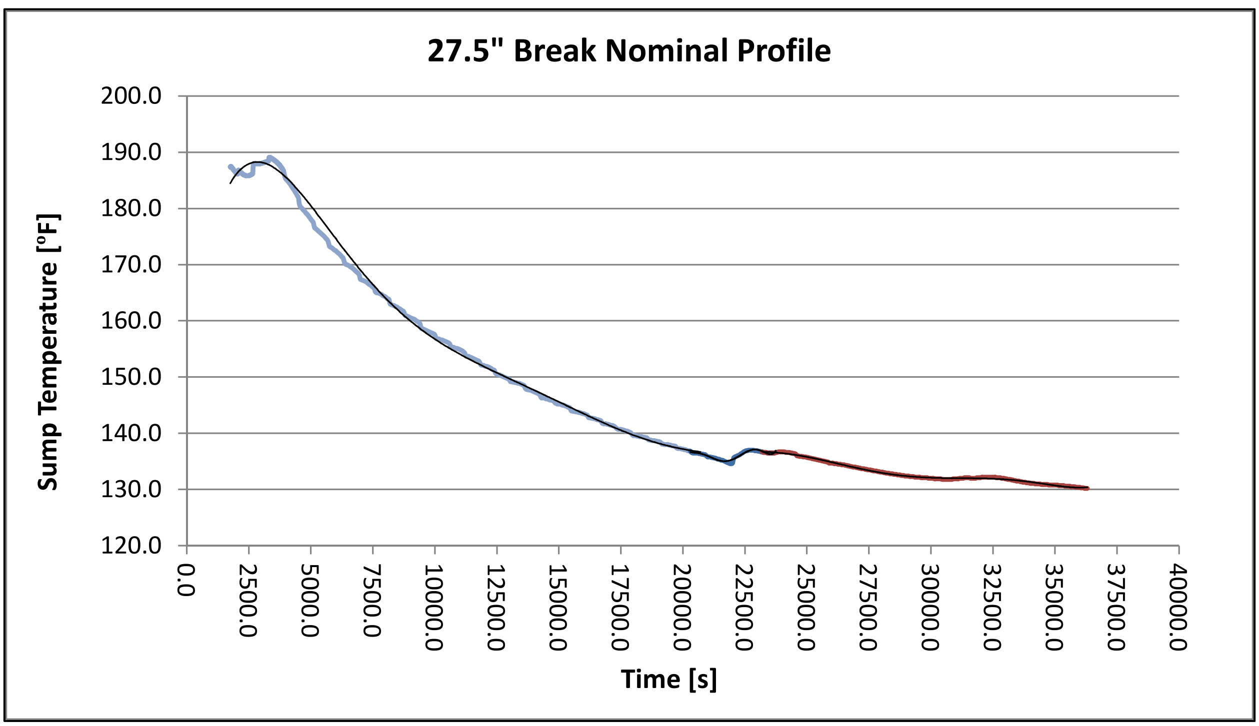

Figure 3. 27.5" Break Nominal Profile Data and Fitted Functions

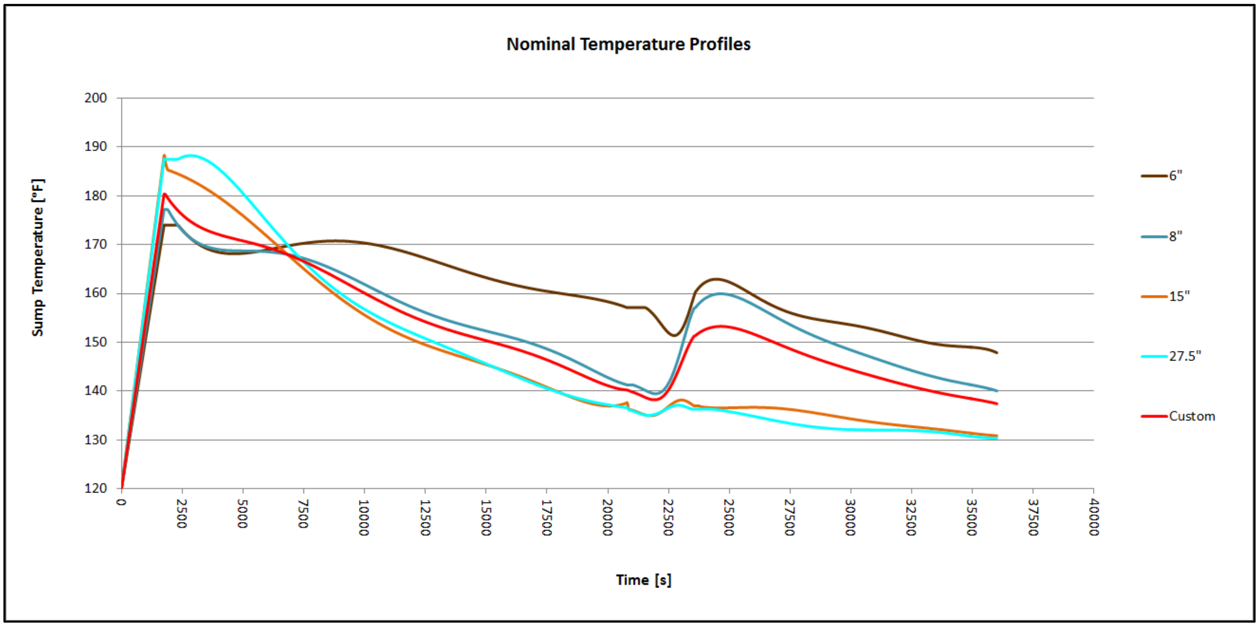

Figure 4. Fit Functions for Nominal Profiles for All Break Sizes for which Data was Provided Along with a Custom Interpolated Profile

Table 4. Fitting Results for Maximum Temperature Profiles

Function	Par.	Coefficients			
		6" Break	8" Break	15" Break	27.5" Break
Startup (T_1)	θ_0	0.014314286	0.013342857	0.017085714	0.0194
	θ_1	120.00	120.00	120.00	120.00
Function 1 (T_2)	θ_0	184.3811	260.6763	274.0537	251.5227
	θ_1	0.0055	-0.0659	-0.0817	-0.0623
	θ_2	-6.7110E-06	1.6256E-05	2.4912E-05	2.0463E-05
	θ_3	1.6000E-09	-1.7403E-09	-3.3529E-09	-2.8948E-09
	θ_4	-1.4994E-13	9.5299E-14	2.3019E-13	2.0672E-13
	θ_5	6.2679E-18	-2.6198E-18	-7.8785E-18	-7.3105E-18
	θ_6	-9.7771E-23	2.8709E-23	1.0666E-22	1.0176E-22
Function 2 (T_3)	θ_0	645608.30	1098484.53	-1816047.22	1553887.45
	θ_1	-173.31038	-292.05925	490.87109	-426.77000
	θ_2	1.93770E-02	3.23130E-02	-5.52260E-02	4.88310E-02
	θ_3	-1.15460E-06	-1.90380E-06	3.31070E-06	-2.97910E-06
	θ_4	3.86710E-11	6.29940E-11	-1.11530E-10	1.02200E-10
	θ_5	-6.90250E-16	-1.10980E-15	2.00200E-15	-1.86950E-15

	θ_6	5.12990E-21	8.13300E-21	-1.49590E-20	1.42450E-20
Function 3 (T_4)	θ_0	76171.82643	73973.96030	-3667.19028	18740.95681
	θ_1	-15.46832	-15.06974	0.81452	-3.91767
	θ_2	1.30530E-03	1.27570E-03	-7.10930E-05	3.43310E-04
	θ_3	-5.84300E-08	-5.72800E-08	3.30760E-09	-1.59480E-08
	θ_4	1.46400E-12	1.43900E-12	-8.63520E-14	4.14230E-13
	θ_5	-1.94690E-17	-1.91830E-17	1.19830E-18	-5.70420E-18
	θ_6	1.07390E-22	1.06030E-22	-6.90240E-24	3.25330E-23

Table 5. R^2 Values for Maximum Temperature Profile Data Fitting

Break Size / R^2	T_2	T_3	T_4
6"	1.0000	0.9990	0.9970
8"	0.9990	0.9980	0.9980
15"	1.0000	0.9970	0.9990
27.5"	1.0000	0.9960	0.9970
Mean	0.9998	0.9975	0.9978
Min	0.9990	0.9960	0.9970

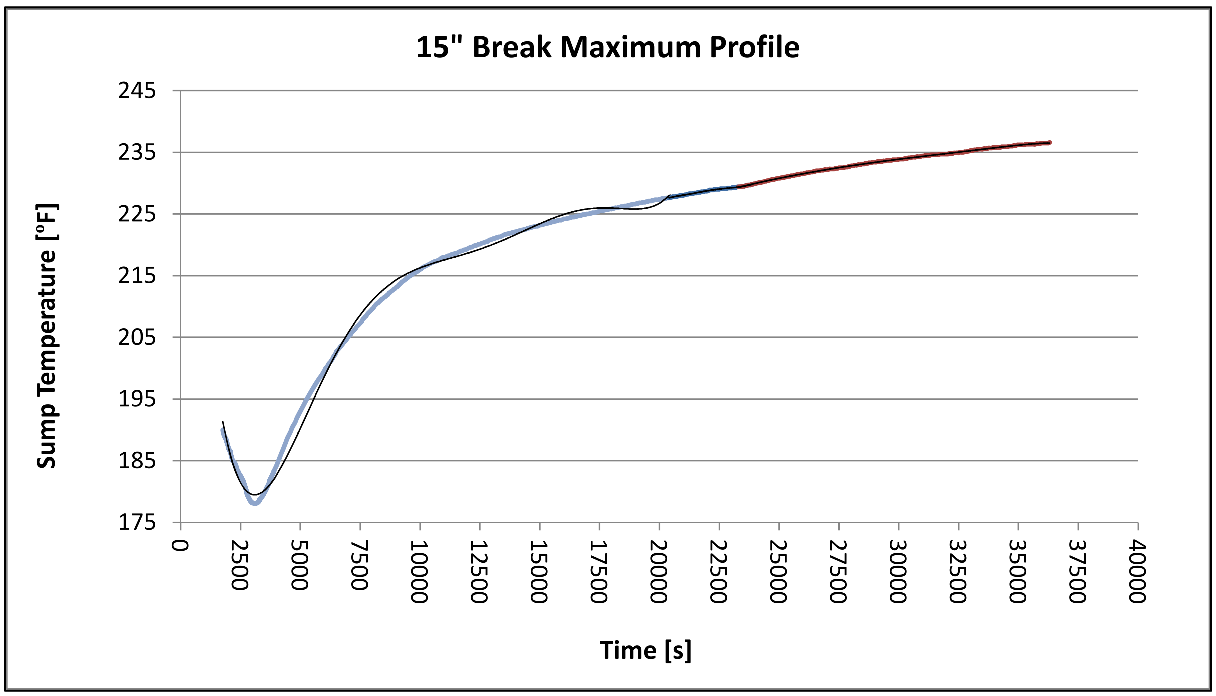

Figure 5. 15" Break Maximum Profile Data and Fitted Functions

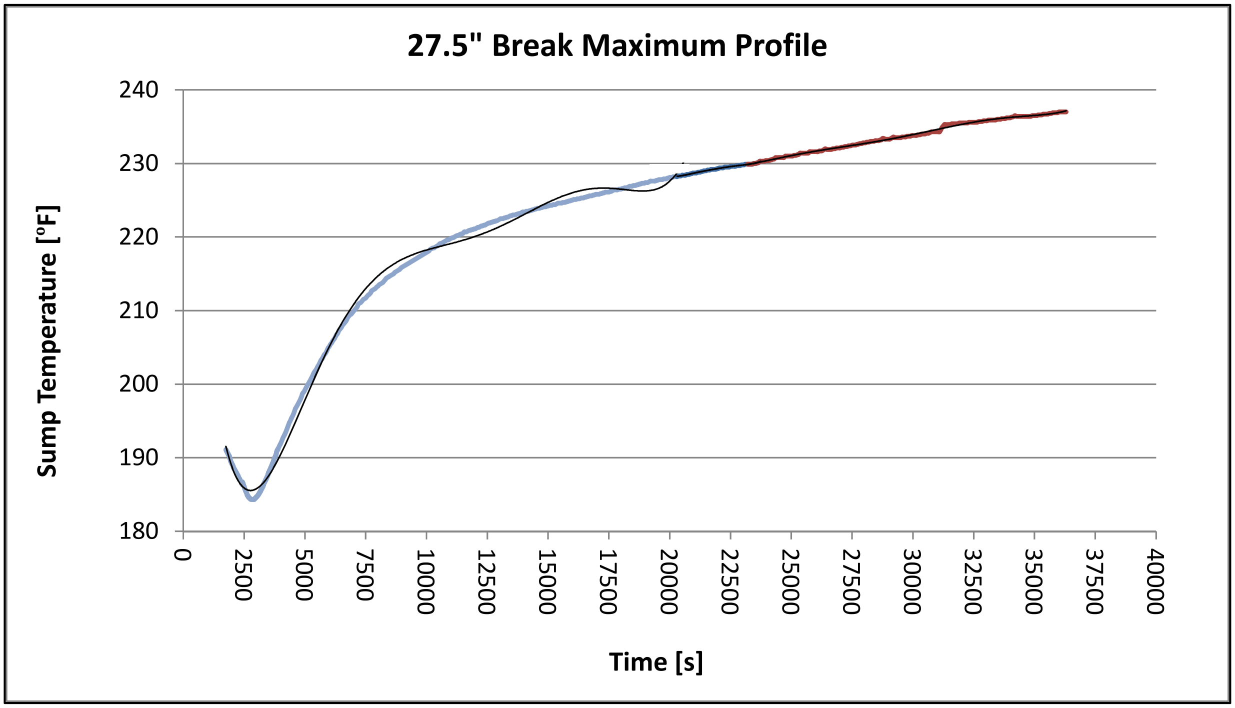

Figure 6. 27.5" Break Maximum Profile Data and Fitted Functions

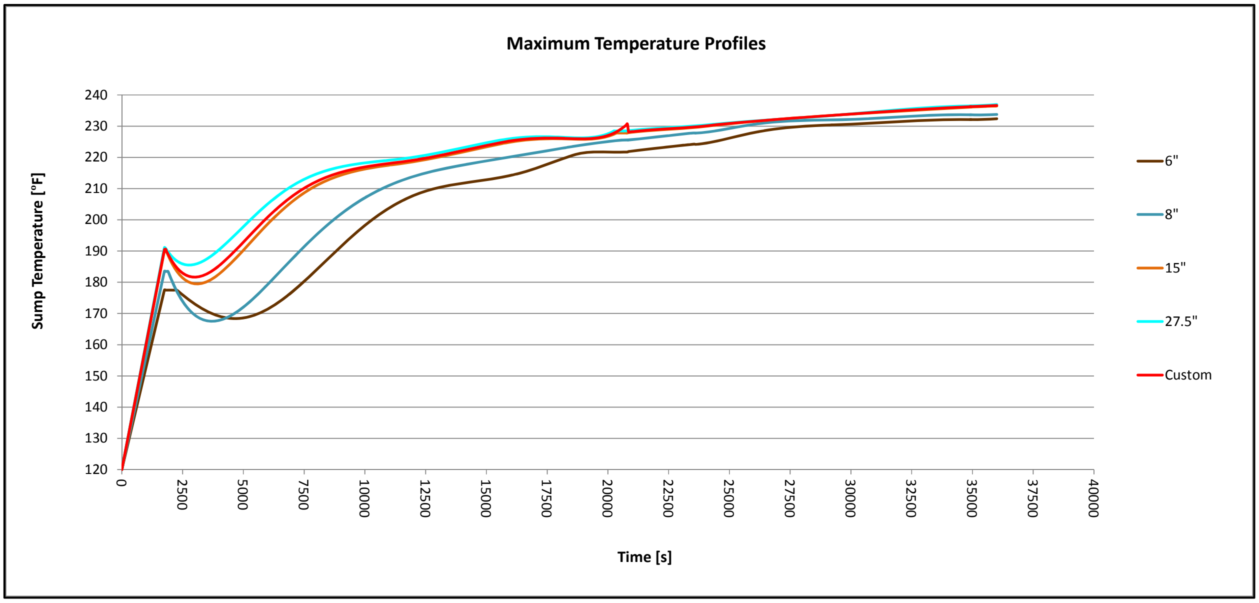

Figure 7. Fit Functions for Maximum Profiles for All Break Sizes for which Data was Provided Along with a Custom Interpolated Profile

Table 6. Fitting Results for Minimum Temperature Profiles

Function	Par.	Coefficients
		6" Break
Startup (T_1)	θ_0	0.013694286
	θ_1	120.00
Function 1 (T_2)	θ_0	215.47370
	θ_1	-0.03185
	θ_2	7.9295E-06
	θ_3	-1.0342E-09
	θ_4	7.2725E-14
	θ_5	-2.6431E-18
	θ_6	3.8772E-23
Function 2 (T_3)	θ_0	10801130.02
	θ_1	-2858.11616
	θ_2	3.14710E-01
	θ_3	-1.84560E-05
	θ_4	6.07970E-10
	θ_5	-1.06660E-14
	θ_6	7.78440E-20
Function 3 (T_4)	θ_0	11921.35
	θ_1	-2.27350
	θ_2	1.82690E-04
	θ_3	-7.80990E-09
	θ_4	1.87110E-13
	θ_5	-2.38120E-18
	θ_6	1.25750E-23

Table 7. R^2 Values for Minimum Temperature Profile Data Fitting

Break Size / R^2	T_2	T_3	T_4
6"	0.9970	0.9970	1.0000

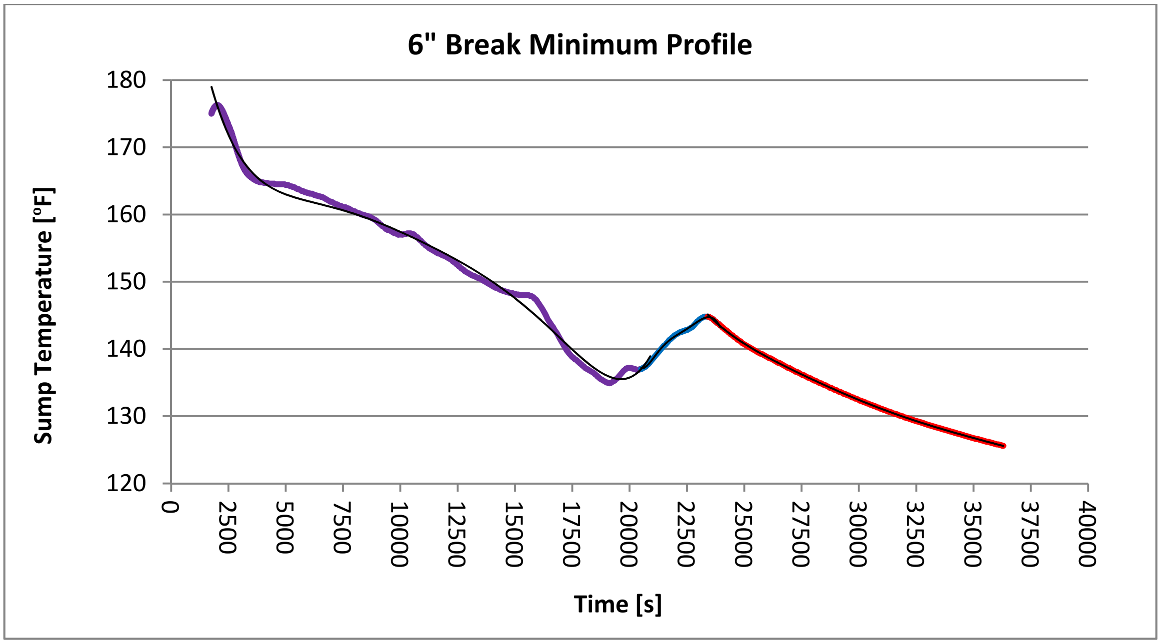

Figure 8. 6" Break Minimum Profile Data and Fitted Function

In order to demonstrate the results of using this methodology, we programmed the algorithm in Microsoft Excel. The figures below show screenshots of custom profiles generated by this method. For a given custom break size, we show the nominal, minimum, and maximum profiles, and use the profile index factor to determine where the actual custom profile lies. Figure 9 shows the custom profile for a 10.75" break and a profile index of 0.50. Figure 10 shows the custom profile for a 19.28" break and a profile index of 0.29. Tables 8 and 9 show the respective input values for each of these profiles.

Table 8. Data Table for 10.75" Break and Profile Index of 0.50

User Input Parameters	User Value	Acceptable Range	
		Min	Max
Custom Break Size (Inches)	10.75	6	27.5
Profile Index	0.5	-1	1
	-1 = Minimum Profile 0 = Nominal Profile 1 = Maximum Profile		

Computed Parameters	Value	Range	
		Min	Max
Break Size Range Coefficients	0.3929	0	1
Break Size Range	10.75	8	15

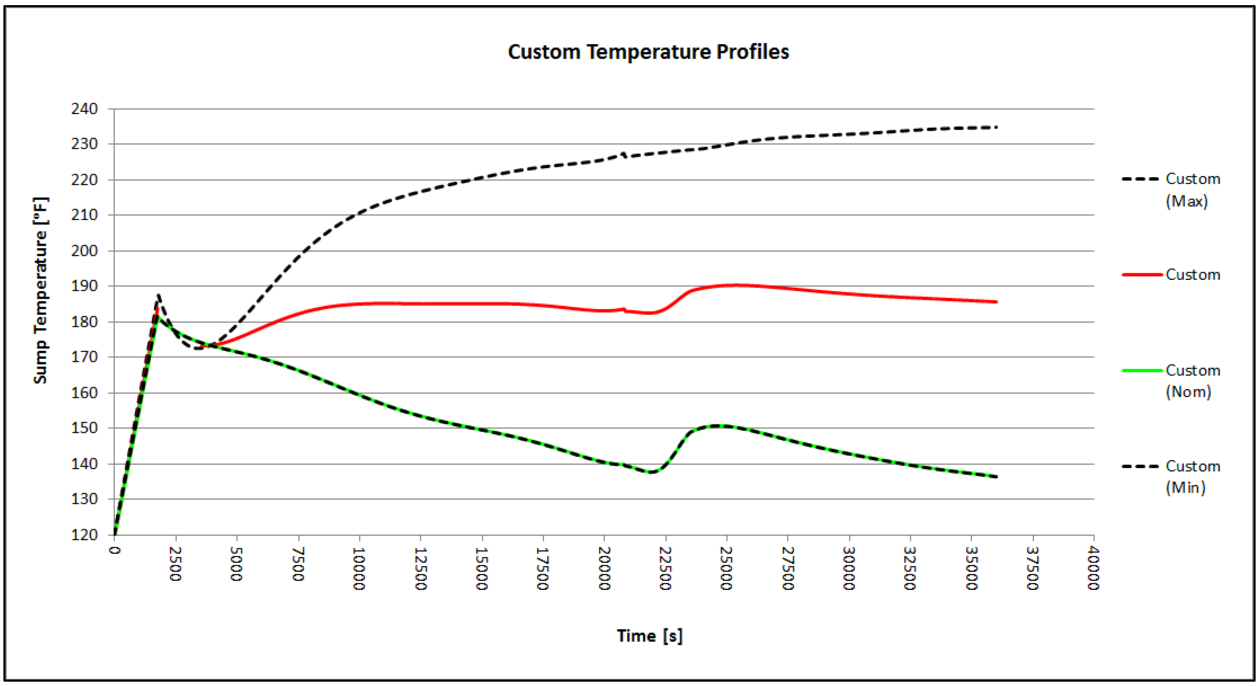

Figure 9. Custom Temperature Profiles for 10.75" Break and Profile Index of 0.50. Note: The Custom (Min), Custom (Max), and Custom (Nom) all correspond to a 10.75" LOCA with *P*= -1, 1, and 0, respectively. The red line denoted Custom corresponds to the profile index of 0.5.

Table 9. Data Table for 19.28" Break and Profile Index of 0.29

User Input Parameters	User Value	Acceptable Range	
		Min	Max
Custom Break Size (Inches)	19.28	6	27.5
Profile Index	0.29	-1	1
	-1 = Minimum Profile 0 = Nominal Profile 1 = Maximum Profile		
Computed Parameters	**Value**	**Range**	
		Min	**Max**
Break Size Range Coefficients	0.3424	0	1
Break Size Range	19.28	15	27.5

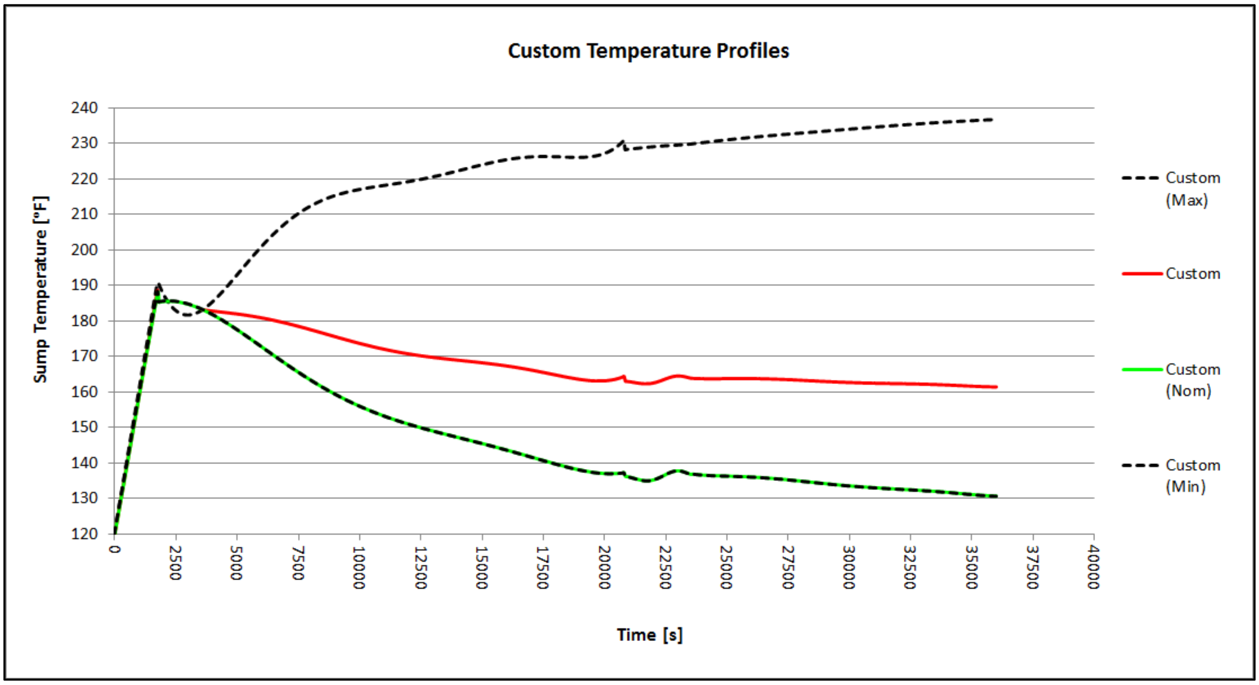

Figure 10. Custom Temperature Profiles for 19.28" Break and Profile Index of *P*=0.29

Lastly, we examine how accurately we can interpolate the 8" nominal profile from the neighboring 6" and 15" nominal profiles by plotting the interpolated 8" profile versus the actual 8" profile in Figure 11. The correlation coefficient between the interpolated 8" profile and the actual 8" profile is 0.8919. The closer the correlation coefficient is to one, the better the fit. In this case, the correlation coefficient is slightly smaller than our lowest R^2 value for a time slice of 0.91. There are several reasons to expect this. First, this is an "out-of-sample test" as opposed to fitting the data directly. Second, the gap between the nominal curves for 6"- and 15"-breaks is significant, particularly at times past 20,000 seconds; see Figure 4. Third, the trend in the nominal temperature profiles as we move from 6" to 8" to 15" to 27.5" breaks in Figure 4 suggests nonlinearity in the changes but we are employing linear interpolation. With this as context, noting that a correlation coefficient of 0.90 or greater indicates relatively high correlation, and noting that the gap between profiles for the 6"- and 15"-breaks tends to be larger than those for which we recommend our actual interpolations (see Figure 4), we regard the correlation coefficient we obtain as large enough to view the fits as reasonable.

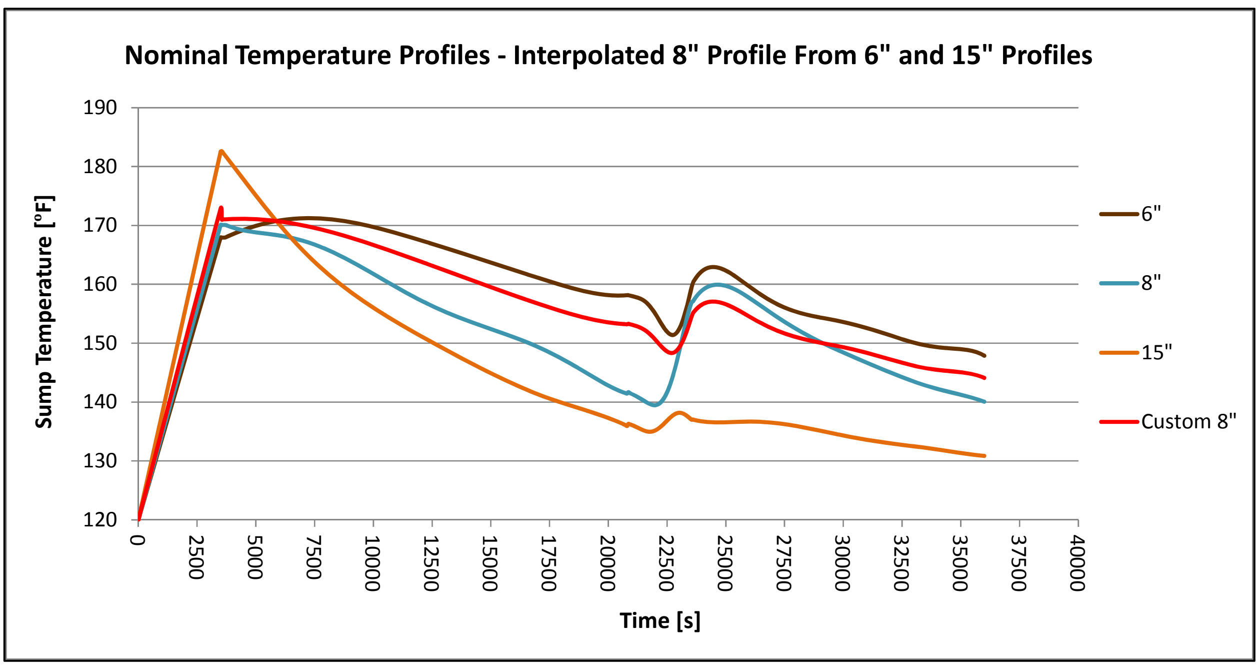

Figure 11. Interpolated 8" Nominal Profile Using 6" Profile and 15" Profile vs. Actual 8" Nominal Profile

5. Conclusions

The method we describe in this report generates a target weighted polynomial function by weighting the coefficients of the neighboring functions appropriately. For this methodology to be valid, all polynomial functions within a time slice must be of the same degree, and the time slices $\{T_1, T_2, T_3, T_4\}$ must be the same across all data and functions. When using six-degree polynomials, care should be taken to carry an appropriate number of decimal places to avoid round off error when programming this method.

The goodness-of-fit values for the each of polynomial functions for all break sizes and time slices are excellent, that is, all of the R^2 values are above 0.90, and many of them are above 0.99. The six degree polynomials provide precise representations of the simulated data.

The underlying assumption is that the break sizes on the continuum from 6" to 27.5" were discretized in such a way that it is reasonable to believe that curves for break sizes between these discrete intervals are weighted averages of their neighbors. In other words, the profile for a 10" break lies between the profile for an 8" break and a 15" break. The nominal profiles for the 15" break and the 27.5" break are very close, so interpolation of large break sizes should in particular be very accurate. A finer discretization would obviously lead to more accurate results.

When implementing this method in the CASA Grande simulation model, one will be able to generate the complete temperature profile for any break size. In addition, the user will be able to control whether the temperature is at the nominal, minimum, or maximum levels as well as any possible level in between. This should enable a powerful and complete sensitivity analysis for a range of temperature-profile scenarios.

7. References

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